Illustrative Example 23.1. *Determine the ultimate bearing capacity of a strip footing, 1.20 m wide, and having the depth of foundation of 1.0 m. Use Terzaghi's theory and assume general shear failure. Take* $\phi' = 35°$, $\gamma = 18\ kN/m^3$, *and* $c' = 15\ kN/m^2$.

Solution. From Eq. 23.25, $q_u = c' N_c + \gamma D_f N_q + 0.5 \gamma B N_\gamma$

For $\phi' = 35°$, Table 23.1 gives $N_c = 57.8$, $N_q = 41.4$ and $N_\gamma = 42.4$.

Now

$$q_u = 15.0 \times 57.8 + 18.0 \times 1.0 \times 41.4 + 0.5 \times 18.0 \times 1.2 \times 42.4$$
$$= \mathbf{2070\ kN/m^2}$$

Illustrative Example 23.2. *Determine the allowable gross load and the net allowable load for a square footing of 2m side and with a depth of foundation of 1.0 m. Use Terzaghi's theory and assume local shear failure. Take a factor of safety of 3.0. The soil at the site has $\gamma = 18\ kN/m^3$, $c' = 15\ kN/m^2$ and $\phi' = 25°$.*

Solution. From Table 23.1, for $\phi' = 25°$

$$N_c' = 14.8,\quad N_q' = 5.6 \quad \text{and} \quad N_\gamma' = 3.2$$

From Eq. 23.37, taking $c_m' = 2/3\ c' = 10\ \text{kN/m}^2$

$$q_u = 1.2 \times 10.0 \times 14.8 + 18 \times 1.0 \times 5.6 + 0.4 \times 18 \times 2 \times 3.2$$
$$= 325\ \text{kN/m}^2$$

From Eq. 23.1, $q_{nu} = 325 - 18 \times 1.0 = 307\ \text{kN/m}^2$

From Eq. 23.2, $q_{ns} = \dfrac{q_{nu}}{F} = \dfrac{307}{3.0} = 102.3\ \text{kN/m}^2$

Net allowable load $= 102.3 \times (2 \times 2) = 409.2\ \text{kN}$

From Eq. 23.3, $q_s = q_{ns} + \gamma D_f = 102.3 + 18 \times 1.0 = 120.3\ \text{kN/m}^2$

Gross allowable load $= 120.3 \times (2 \times 2) = \mathbf{481.2\ kN}$

Illustrative Example 23.3. *A footing 2 m square is laid at a depth of 1.3 m below the ground surface. Determine the net ultimate bearing capacity using IS code method. Take $\gamma = 20\ kN/m^3$, $\phi' = 30°$ and $c' = 0$.*

Solution. For $\phi' = 30°$, Table 23.6 gives

$$N_c = 30.14,\quad N_q = 18.4 \quad \text{and} \quad N_\gamma = 22.4$$

From Table 23.3, $s_c = 1.3$, $s_q = 1.2$ and $s_\gamma = 0.80$

From Eq. 23.49 (*a*), $d_c = 1 + 0.2\,(D_f/B)\tan(45° + \phi'/2)$
$= 1 + 0.2 \times (1.3/2.0)\tan 60° = 1.23$

From Eq. 23.49 (*c*), $d_q = d_\gamma = 1 + 0.1\,(D_f/B)\tan(45° + \phi'/2) = 1.11$

From Eq. 23.48, $q_{nu} = cN_c\,s_c\,d_c\,i_c + q\,(N_q - 1)\,s_q\,d_q\,i_q + 0.5\,\gamma\,B\,N_\gamma\,s_\gamma\,d_\gamma\,i_\gamma W'$

$$= 0.0 + 1.3 \times 20 \times (18.4 - 1) \times 1.2 \times 1.11 \times 1.0 + 0.5 \times 20 \times 2.0 \times 22.4 \times 0.8 \times 1.11 \times 1.0$$

or $q_{nu} = \mathbf{1000\ kN/m^2}$

Illustrative Example 23.4. *Determine the net ultimate bearing capacity of the footing in Illustrative Example 23.3 if*

(a) the water table rises to the level of the base,

(b) the water table rises to the ground surface, and

(c) the water table is 1 m below the base.

Solution: (*a*) $W' = 0.50$, Therefore, Eq. 23.48 gives

$$q_{nu} = 1.3 \times 20.0 \times (18.4 - 1) \times 1.2 \times 1.11 \times 1.0 + 0.5 \times 20.0 \times 2.0 \times 22.4 \times 0.8 \times 1.11 \times 0.5$$
$$= \mathbf{801\ kN/m^2}$$

(*b*) $W' = 0.50$. The surcharge q is also reduced as the effective stress is reduced, Thus

$$q_{nu} = 1.3 \times (20 - 9.81) \times (18.4 - 1) \times 1.2 \times 1.11 \times 1.0 + 0.5 \times 20 \times 2.0 \times 22.4 \times 0.8 \times 1.11 \times 0.5$$
$$= \mathbf{506\ kN/m^2}$$

(c) W' is obtained by linear interpolation [see Eq. 23.36 (c)].

$$W' = 0.5 + \frac{0.5 \times 1.0}{2.0} = 0.75$$

Therefore,

$$q_{nu} = 1.3 \times 20.0 \times (18.4 - 1) \times 1.2 \times 1.11 \times 1.0 + 0.5 \times 20.0 \times 2.0 \times 22.4 \times 0.8 \times 1.11 \times 0.75$$
$$= 901 \text{ kN/m}^2$$

Illustrative Example 23.5. *A square column foundation is to be designed for a gross allowable total load 250 kN. If the load is inclined at an angle of 15° to the vertical, determine the width of the foundation. Take a factor of safety of 3.0 and use Vesic's equation. γ = 19 kN/m³, ϕ' = 35°, and c' = 5 kN/m². The depth of foundation is 1.0 m.*

Solution. From Eq. 23.45,

$$q_u = c' N_c s_c d_c i_c + qN_q s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

From Table 23.6, $N_c = 46.12$, $N_q = 33.30$ and $N_\gamma = 48.03$

From Table 23.7,

$$s_c = 1 + 33.30/46.12 = 1.72$$
$$s_q = 1 + \tan 35° = 1.70, \quad s_\gamma = 0.60$$

From Eq. 23.46 (a), $d_c = 1 + 0.4 \times 1.0/B$

From Eq. 23.46 (b), $d_q = 1 + 2 \tan 35° \ (1 - \sin 35°)^2 \times 1.0/B$

or $d_q = 1 + 0.255/B; \quad d_\gamma = 1.0$

From Eq. 23.47 (a), $i_c = i_q = (1 - \alpha°/90)^2 = 0.694$

From Eq. 23.47 (b), $i_\gamma = (1 - \alpha°/\phi)^2 = 0.327$

Therefore,

$$q_u = 5.0 \times 46.12 \times 1.72 \times (1 + 0.4/B) \times 0.694 + (19 \times 1.0)\, 33.3 \times 1.7 \times (1 + 0.255/B) \times 0.694 + 0.5 \times 19 \times B \times 48.03 \times 0.6 \times 1.0 \times 0.327$$
$$= 1022.2 + 300.7/B + 89.5\, B$$

From Eq. 23.1,

$$q_{nu} = q_u - \gamma D_f = q_u - 19 \times 1.0$$
$$= 1003.2 + \frac{300.7}{B} + 89.5\, B$$

From Eq. 23.3,

$$q_s = \frac{q_{nu}}{3} + 19 \times 1.0$$
$$= 334.4 + \frac{100.2}{B} + 29.8\, B + 19.0$$

Now gross load $= q_s \times B^2$

or

$$250.0 = 353.4\, B^2 + 100.2\, B + 29.8\, B^3$$

Solving by trial and error, $B = \mathbf{0.7}$ **m**

Illustrative Example 23.6. *Determine the ultimate bearing capacity of a square footing 2 m × 2m in a soil with unit weight of 18 kN/m³, ϕ' = 20°, c = 20 kN/m². Take the depth of foundation of 1.50 m. Use Hansen's equation.*

Solution. From Eq. 23.42,

$$q_u = cN_c s_c d_c i_c + qN_q s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

From Table 23.2, $N_c = 14.83$, $N_q = 6.40$ and $N_\gamma = 3.54$

From Table 23.3, $s_c = 1.2$, $s_q = 1.2$ and $s_\gamma = 0.6$

From Table 23.4,

$$d_c = 1 + 0.3 \times 1.5/2.0 = 1.225$$
$$d_q = d_c = 1.225, \quad d_\gamma = 1.0$$

As $i_c = i_c = i_\gamma = 1.0$,

$$q_u = 20.0 \times 14.83 \times 1.2 \times 1.225 \times 1.0 + (18 \times 1.50) \times 6.40 \times 1.225 \times 1.0 + 0.5 \times 18 \times 2 \times 3.54 \times 0.6 \times 1.0 \times 1.0$$

$$= \mathbf{728.25\ kN/m^2}$$

Illustrative Example 23.7. *A strip footing of 2 m width is founded at a depth of 4 m below the grou surface. Determine the net ultimate bearing capacity, using (a) Terzaghi's equation, (b) Skempton's equatio and (c) IS Code. The soil is clay ($\phi = 0$, $c = 10\ kN/m^2$). The unit weight of the soil is 20 kN/m³.*

Solution. (*a*) Terzaghi's equation

From Eq. 23.25, $q_u = c_u N_c + \gamma D_f N_q + 0.5 \gamma B N_\gamma$

Taking the values from Table 23.1, $q_u = 10 \times 5.7 + 20 \times 4 \times 1.0 + 0.5 \times 20 \times 2 \times 0.0 = 137$

Therefore, $q_{nu} = q_u - \gamma D_f = 137.0 - 20 \times 4 = \mathbf{57.0\ kN/m^2}$

(*b*) Skempton's equation

From Eq. 23.53 (*a*), for $\frac{D_f}{B} = \frac{4}{2} < 2.5$,

$$N_c = 5.0\,(1 + 0.2\,D_f/B)\,(1 + 0.2\,B/L)$$

$$= 5.0\,(1 + 0.2 \times 4/2)\,(1 + 0.2 \times 0.0) = 7$$

From Eq. 23.55, $q_{nu} = c_u N_c = 10 \times 7.0 = \mathbf{70.0\ kN/m^2}$

(*c*) IS Code

From Eq. 23.56, $q_{nu} = c_u N_c s_c d_c i_c$

Taking $N_c = 5.14$, $q_{nu} = 10 \times 5.14 \times \left(1 + 0.2 \times \left(\frac{D_f}{B}\right) \tan 45°\right) \times 1.0$

$$= 51.4 \times 1.4 = \mathbf{71.96\ kN/m^2}$$

Illustrative Example 23.8. *A square footing (1.5 m × 1.5 m) is located at a depth of 1.0 m in a clay deposit consisting of two layers. The top layer is 1m thick and has $c_1 = 150\ kN/m^2$ and $\gamma_1 = 16\ kN/m^3$. The bottom layer has $c_2 = 50\ kN/m^2$ and $\gamma = 15\ kN/m^2$. Determine the net ultimate bearing capacity.*

Solution. From Eq. 23.59, taking $i_c = 1.0$, $q_u = c_1 N_c s_c d_c + q$

From Fig. 23.18, for $c_2/c_1 = 1/3$ and $Z/B = 1.0/1.5 = 0.67$, the value of N_c is equal to 3.50.

$$s_c = 1 + (B/L)\,(N_q/N_c) = 1 + 1 \times 1/3.5 = 1.29$$

$$d_c = 1 + 0.4 \times 1.0/1.50 = 1.27$$

Therefore, $q_u = 150 \times 3.5 \times 1.29 \times 1.27 + 16.0 \times 1.0 = 876.1\ kN/m^2$

$$q_{nu} = 876.1 - 16 = \mathbf{860.1\ kN/m^2}$$

Illustrative Example 23.9. *A square footing (1.5 m × 1.5 m) is located at a depth of 1.0 m. The footing is subjected to an eccentric load of 400 kN, with an eccentricity of 0.2 m along one of the symmetrical axes. Determine the factor of safety against bearing failure. Use Vesic's equation. Take $\gamma = 21\ kN/m^3$, $c = 100\ kN/m^2$, $\phi = 0$.*

Solution. Effective width $B' = B - 2\,e_b = 1.5 - 2 \times 0.2 = 1.1$ m

From Eq. 23.45, taking $N_\gamma = 0.0$, $N_c = 5.14$ and $N_q = 1.0$,

$$q_u = cN_c s_c d_c i_c + q\,N_q s_q d_q i_q$$

where $s_c = 1 + (B'/L)\,(N_q/N_c) = 1 + (1.1/1.50) \times 1.0/5.14 = 1.14$

$s_q = 1 + (B'/L) \tan\phi = 1.0 + (1.1/1.50) \tan 0° = 1.00$

$$d_c = 1 + 0.4\,(D_f/B) = 1 + 0.4 \times 1.0/1.5 = 1.27$$

$$d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2\,(D_f/B) = 1.0$$

Therefore,

$$q_u = 100 \times 5.14 \times 1.14 \times 1.27 + 1.0 \times (21.0 \times 1.0) \times 1.0 \times 1.0$$

$$= 744.2 + 21.0 = 765.2 \text{ kN/m}^2$$

From Eq. 23.63 (*a*),

$$q_{max} = \frac{Q}{BL}\left(1 + \frac{6e}{B}\right) = \frac{400}{1.5 \times 1.5}\left(1 + \frac{6 \times 0.2}{1.5}\right)$$

or

$$q_{max} = 320 \text{ kN/m}^2 \qquad \ldots(a)$$

Now

$$q_{nu} = 765.2 - 21 \times 1 = 744.2.$$

Therefore,

$$q_s = \left(\frac{744.2}{F} + 21.0\right) \qquad \ldots(b)$$

From Eqs. (*a*) and (*b*), $744.2/F + 21.0 = 320$

or

$$F = \mathbf{2.49.}$$

Illustrative Example 23.10. *A square footing is required to carry a net load of 1200 kN. Determine the ... of the footing if the depth of foundation is 2 m and the tolerable settlement is 40 mm. The soil is sandy ... N = 12. Take a factor of safety of 3.0. The water table is very deep. Use Teng's equation.*

Solution. From Eq. 23.61,

$$q_{nu} = 0.33\,N^2 B W_\gamma + 1.0(100 + N^2)\,D_f\,W_q$$

or

$$q_{nu} = 0.33\,(12)^2 B \times 1.0 + 1.00\,(100 + 12^2) \times 2 \times 1.0$$

or

$$q_{nu} = 47.5\,B + 488.0$$

Total net load, $Q_n = (47.5\;B + 488.0)/3 \times B^2$

or

$$1200 = (47.5B + 488.0)/3 \times B^2$$

or

$$1200 = 15.8\,B^3 + 162.7\,B^2$$

Solving, by trial and error, $B = 2.45$ m.

From Eq. 23.80 (*b*),

$$q_{np} = 1.40\,(N - 3)\left(\frac{B + 0.3}{2\,B}\right)^2 W_\gamma R_d\, s$$

or

$$q_{np} = 1.40\,(N - 3)\left(\frac{B + 0.3}{2\,B}\right)^2\left(1 + \frac{2}{B}\right) \times 40$$

or

$$q_{np} = 1.40\,(12 - 3)\left(\frac{B + 0.3}{2\,B}\right)^2 \times 40\;(1 + 0.4/B)$$

or

$$= 126\;(B + 0.3)^2 \times (1 + 0.4/B)$$

Now

$$Q_n = q_{np} \times B^2$$

or

$$1200 = 126\,(B + 0.3)^2\,(1 + 2/B)$$

Solving,

$$B = 1.90 \text{ m. Adopt } B = \mathbf{2.0\ m}$$

Illustrative Example 23.11. *A rectangular footing (3 m × 2 m) exerts a pressure of 100 kN/m² on a ...esive soil ($E_s = 5 \times 10^4$ kN/m² and $\mu = 0.50$). Determine the immediate settlement at the centre, assuming ... the footing is flexible, (b) the footing is rigid.*

Solution. From Eq. 23.68,

$$s_i = qB\left(\frac{1 - \mu^2}{E_s}\right) I$$

As $L/B = 3/2 = 1.5$, from Table 23.8, $I = 1.36$.

Therefore, $$s_i = 100 \times 2 \left(\frac{1 - 0.5^2}{5 \times 10^4} \right) \times 1.36 \times 10^{-3} = 4.08 \text{ mm}$$

(b) For rigid footing ($I = 1.06$), $$s = (1.06/1.36) \times s_i$$
$$= 1.06/1.36 \times 4.08 = \mathbf{3.18\ mm}$$

Illustrative Example 23.12. *Fig. E-23.12 shows a square footing resting on a sand deposit. The pressu at the level of the foundation ($\bar{q}$) is 200 kN/m². The figure also shows the variation of the elastic moduli with depth. Determine the settlement of the foundation after 6 years of construction.*

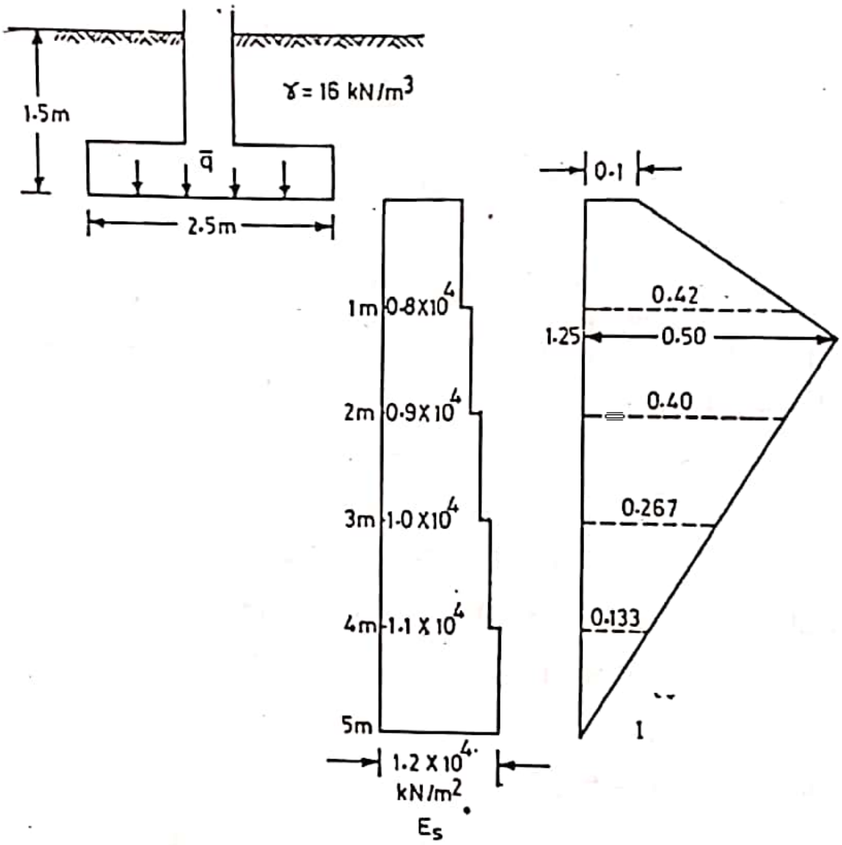

Fig. E-23.12.

Solution. From Eq. 23.69, $$s_i = C_1 C_2 (\bar{q} - q) \sum_0^{2B} \frac{I_z}{E_s} \cdot \Delta z$$

$q = 16 \times 1.5 = 24 \text{ kN/m}^2$ and $\bar{q} - q = 200 - 24 = 176 \text{ kN/m}^2$

$$C_1 = 1 - 0.5 \left(\frac{q}{\bar{q} - q} \right) = 1 - 0.5 (24/176) = 0.932$$

$$C_2 = 1 + 0.2 \log_{10} (t/0.1) = 1 + 0.2 \log_{10} (6/0.1) = 1.356$$

Therefore, $$s_i = 0.932 \times 1.356 \times 176 \sum_0^{2B} \frac{I_z}{E_s} \cdot \Delta z$$

$$= 222.4 \sum_0^{2B} \frac{I_z}{E_s} \cdot \Delta z$$

The value of $\sum_0^{2B} (I_z/E_s) \cdot \Delta z$ is determined as shown in the table below. It is equal to 13.97×10^{-5}.

z	Δz	E_s (kN/m^2)	I_z	$(I_z/E_s) \cdot \Delta z$
0—1.00	1.0 m	8000	$\frac{0.1 + 0.42}{2} = 0.26$	3.25×10^{-5}
1.0—2.0	"	9000	0.453	5.03×10^{-5}
2.0—3.0	"	10000	0.333	3.33×10^{-5}
3.0—4.0	"	11000	0.200	1.82×10^{-5}
4.0—5.0	"	12300	0.067	0.54×10^{-5}
				$\Sigma\ 13.97 \times 10^{-5}$

Therefore, $s_i = 222.4 \times 13.97 \times 10^{-5}$ m

or $s_i =$ **31.07 mm**

Illustrative Example 23.13. *Fig. E-23.13 shows the load- settlement curve obtained from a plate load test conducted on a sandy soil. The size of the plate used was 0.3 m × 0.3 m. Determine the size of a square column footing to carry a net load of 3000 kN with a maximum settlement of 25 mm.*

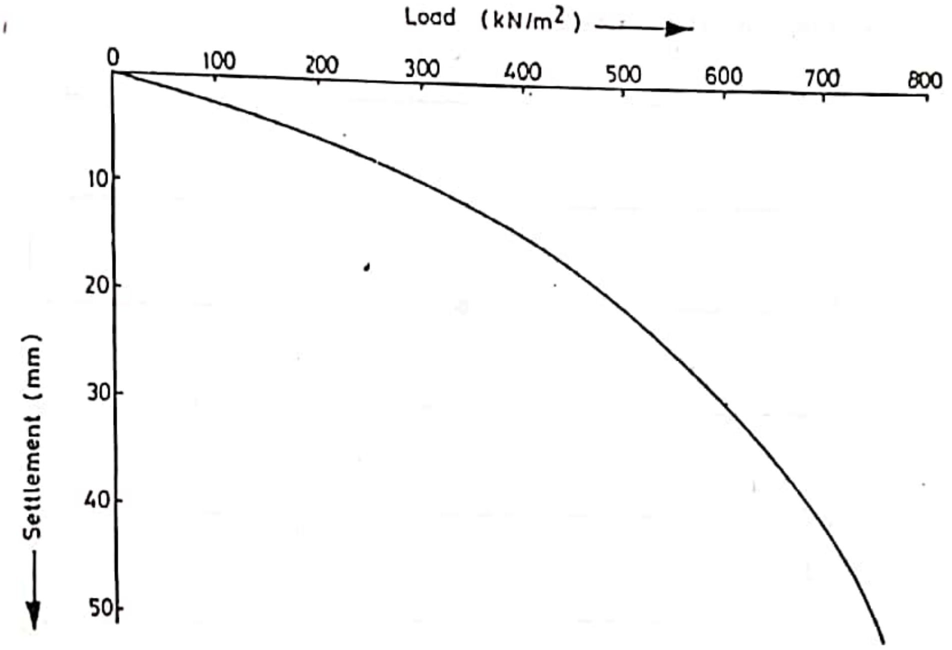

Fig. E-23.13.

Solution. From Eq. 23.88,

$$s_f = s_p \left[\frac{B_f (B_p + 0.3)}{B_p (B_f + 0.3)} \right]^2$$

or

$$s_f = s_p \left(\frac{B_f}{B_p} \right)^2 \left(\frac{0.6}{B_f + 0.3} \right)^2$$

The value of B_f is found by trial and error, as shown in the table below.

B_f	$q_0 = \frac{Q}{(B_f)^2}$	s_p from Fig. Ex. 23.13	B_f/B_p	s_f from Eq. (a)
3.80 m	207.7	6 mm	12.67	20.62 mm
3.6 m	231.5	7 mm	12.00	23.85 mm
3.55 m	238.0	7.3 mm	11.83	24.81 mm

Adopt a size of 3.55 m × 3.55 m.

Illustrative Example 23.14. *Two-plate load tests at a site gave the following results.*

Size of plate	*Load*	*Settlement*
0.305 × 0.305 m	*40 kN*	*25 mm*
0.61 × 0.61 m	*40 kN*	*15 mm*

(a) Assuming Poisson's ratio as 0.3, determine the deformation modulus of the soil.

(b) If there are two columns, one of the size 2.5 m × 2.5 m, carrying a load of 2700 kN, and the other of size 3m × 3m, carrying a load of 3900 kN, determine the differential settlement. The columns are 7 m apart.

Solution. (*a*) For the first test,
$$q_1 = \frac{40}{0.305 \times 0.305} = 430 \text{ kN/m}^2$$
$$q_1 B_1 = 430 \times 0.305 = 131.1 \text{ kN/m}^2$$

For the second test,
$$q_2 = \frac{40}{0.61 \times 0.61} = 107.5 \text{ kN/m}^2$$
$$q_2 B_2 = 107.5 \times 0.61 = 65.6 \text{ kN/m}^2$$

Fig. E-23.14 shows the plot between qB and s.

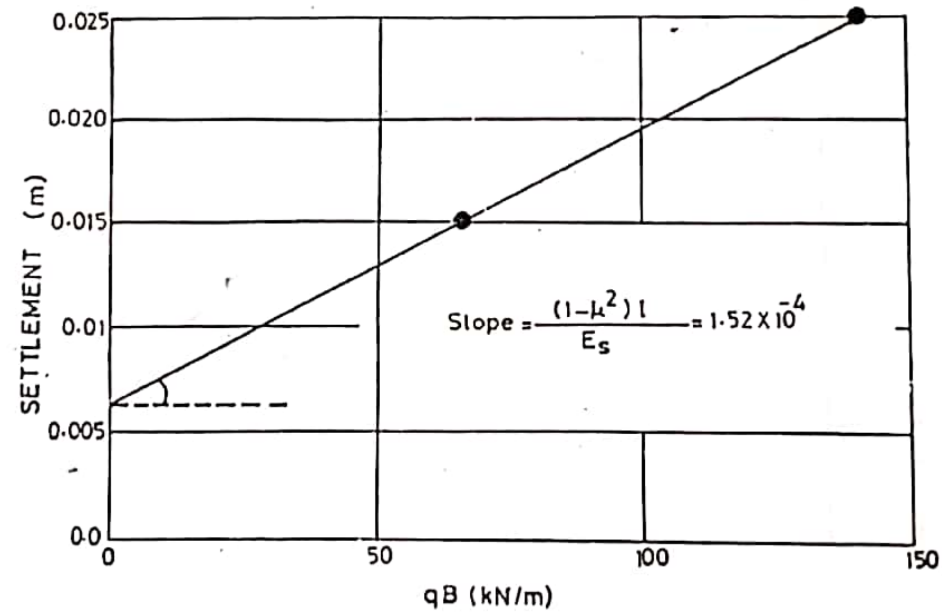

Fig. E-23.14.

From the plot,
$$\left(\frac{1-\mu^2}{E_s}\right) I = 1.52 \times 10^{-4}$$

From Table 23.8,
$$I = 1.12$$

As the plate is rigid,
$$I = 0.8 \times 1.12 = 0.89$$

Therefore,
$$E_s = \frac{(1-\mu^2) \times 0.896}{1.52 \times 10^{-4}} = \frac{(1-0.3^2)}{1.52 \times 10^{-4}} \times 0.896$$
$$E_s = 5364 \text{ kN/m}^2$$

(*b*) For the first column,
$$q_1 = \frac{2700}{2.5 \times 2.5} = 432 \text{ kN/m}^2$$

For the second column,
$$q_2 = \frac{3900}{3 \times 3} = 433 \text{ kN/m}^2$$

As the settlement of the plate (0.305 m × 0.305 m) at a load intensity of 430 kN/m^2 is 25 mm, it can be sed for the determination of the settlement of columns.

Form Eq. 23.88,

$$(s_f)_1 = 25\left[\frac{2.5 \times (0.305 + 0.30)}{0.305 \times (2.5 + 0.3)}\right]^2 = 78.42 \text{ mm}$$

$$(s_f)_2 = 25\left[\frac{3\,(0.305 + 0.30)}{0.305\,(3.0 + 0.3)}\right]^2 = 81.3 \text{ mm}$$

Differential settlement = 81.3 − 78.42 = **2.88 mm**

Illustrative Example 23.15. *The results of two plate load tests for a settlement of 25.4 mm are given.*

Plate diameter	*Load*
0.305 m	31 kN
0.61 m	65 kN

A square column foundation is to be designed to carry a load of 800 kN with an allowable settlement of 25.4 mm. Determine the size using Housel's method.

Solution. From Eq. 23.91 and 23.92,

$$31.0 = (\pi/4) \times (0.305)^2 \times m + \pi\,(0.305) \times n \qquad \ldots(a)$$

$$65.0 = (\pi/4) \times (0.61)^2 \times m + \pi.\,(0.61) \times n \qquad \ldots(b)$$

Eq.(*a*) can be written as

$$62 = 2 \times \pi/4\;(0.305)^2 \times m + 2\pi\,(0.305) \times n \qquad \ldots(c)$$

From Eqs.(*b*) and (*c*), by subtraction,

$$3.0 = m\,[\pi/4\,(0.372 - 0.186)] \quad \text{or} \quad m = 20.55$$

From Eq. (*a*)

$$31.0 = 1.5 + 0.9577\,n \quad \text{or} \quad n = 30.80$$

From Eq. 23.93,

$$Q = B^2 \times 20.55 + (30.8 \times 4B)$$

or

$$800 = 20.55\,B^2 + 123.2\,B$$

or

$$B = 3.93 \text{ m say } \mathbf{4\,m \times 4\,m.}$$