Control Chart for Mean

SESSION 1

Structure

1.1 INTRODUCTION

In Unit 2 of MSTE-001 (Industrial Statistics-I), you have learnt that control charts for variables are used to control the measurable quality characteristics. We use the control chart for the process mean, i.e., $\bar{X}$-chart for controlling the mean of the quality characteristic or process mean. With the help of $\bar{X}$-chart, we monitor the variation in the mean of the samples that have been drawn from time to time from the process.

Prerequisite

- Lab Sessions 1, 3 and 6 of MSTL-001 (Basic Statistics Lab).
- Unit 2 of MSTE-001 (Industrial Statistics-I).

We can use control charts for mean in both cases: When process variability is known and when it is unknown (see Fig. 1.1). You have also learnt in Sec. 2.4 of Unit 2 that we estimate the unknown process variability with the help of sample range or standard deviation.

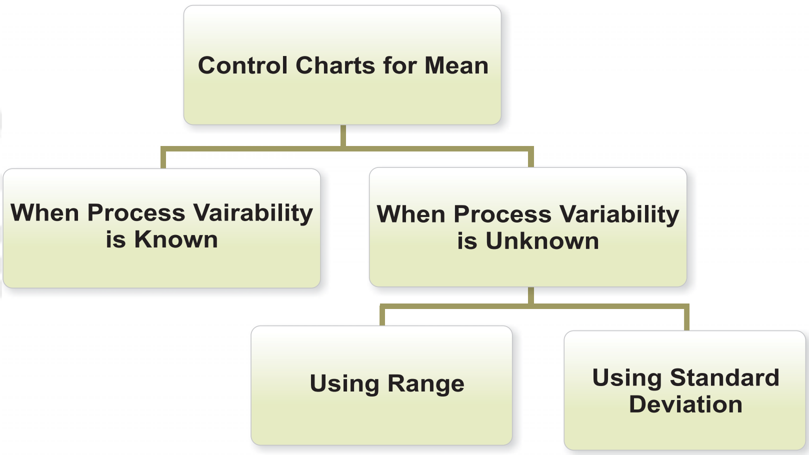


Fig. 1.1

In this lab session, you will learn how to prepare the control chart for mean in MS Excel 2007 **when process variability is known**. We shall illustrate it using a specific problem. You will learn how to prepare the control chart for mean for unknown process variability in Lab Sessions 2 and 3.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for mean;
- construct the control chart for mean; and
- interpret the control chart for mean.

1.2 PROBLEM DESCRIPTION

In this lab session, we consider the problem related to a manufacturing process for maintaining the quality of bottling procedure. A fruit juice manufacturing company uses automatic machines to fill 500 ml juice bottles. A quality control inspector at this company collected 25 samples of four observations of the juice bottles at four different times and measured the volume of each filled bottle. The data is given in Table 1.

Table 1: Volume of fruit juice in the filled bottles

Sample Number	Volume of Juice per Bottle (in ml)			
	Obs. 1	Obs. 2	Obs. 3	Obs. 4
1	497.32	500.62	498.68	497.82
2	504.76	500.00	498.32	500.32
3	499.24	497.18	498.12	498.68
4	499.26	496.32	498.88	497.82
5	502.32	503.62	504.56	503.12
6	502.12	500.32	501.38	500.94
7	499.34	498.32	497.32	497.62
8	499.38	498.12	500.62	498.12
9	501.26	502.38	500.68	501.38
10	498.60	497.62	499.25	498.56
11	502.44	500.00	501.32	499.38
12	501.26	502.32	500.76	502.68
13	497.32	498.50	497.18	499.38
14	499.56	498.00	498.76	501.12
15	502.24	500.32	503.12	501.25
16	501.76	500.50	502.68	501.12
17	500.65	497.82	494.06	496.25
18	501.12	501.26	500.44	502.76
19	501.00	500.50	501.56	501.76
20	497.50	498.82	499.76	497.82
21	503.44	500.62	500.00	501.26
22	499.38	498.38	497.56	498.56
23	501.56	499.56	498.00	499.82
24	498.32	497.32	499.56	498.62
25	499.50	501.12	502.50	500.38

The quality control inspector of the company needs to develop $\overline{X}$-chart to check whether the process of bottling is under control or out-of-control when process variability is known.

Therefore, the problem for this session is to construct the control chart for mean using the data given in Table 1. It is given that the standard deviation for the bottling operation is 2.5 ml.

1.3 PROCEDURE FOR THE CONSTRUCTION OF $\bar{X}$-CHART

You have learnt in Unit 2 of MSTE-001 that the $\bar{X}$-chart (average control chart) is used to monitor the changes in the average value of a quality characteristic. This helps us to detect whether the process is under control or out-of-control. The main steps involved in the construction of $\bar{X}$-chart for any process are as follows:

Step 1: We select k samples (subgroups) randomly from the process at different times, each of size n and measure the characteristic of interest.

Step 2: Suppose, the quality characteristic, say, X is measured from each unit of the sample and $X_{i1}, X_{i2}, ..., X_{in}$ are the measurements of the units of the i^{th} sample of size n. Then we calculate the sample mean for each sample, say, $\bar{X}_1, \bar{X}_2, ..., \bar{X}_k$ where $\bar{X}_i$ is the sample mean for i^{th} sample given by

$$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \qquad ...(1)$$

Step 3: We find the mean of all sample means:

$$\bar{\bar{X}} = \frac{1}{k}\sum_{i=1}^{k} \bar{X}_i \qquad ...(2)$$

Step 4: To construct the $\bar{X}$-chart when process variability is known, we calculate the control limits given below:

- ✓ Centre line $(CL) = \bar{\bar{X}}$ ...(3)
- ✓ Upper control limit (UCL) = $\bar{X} + \frac{3\sigma}{\sqrt{n}} = \bar{X} + A\sigma$ ...(4)
- ✓ Lower control limit (LCL) = $\bar{X} - \frac{3\sigma}{\sqrt{n}} = \bar{X} - A\sigma$ ...(5)

where

n – number of observations in each sample,

k – total number of samples,

σ – process standard deviation, and

$A = 3/\sqrt{n}$ is a constant, which depends on the size of the sample.

It is tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 5: After setting the centre line and control limits, we construct the $\bar{X}$-chart by taking sample numbers on the X-axis and sample means on the Y-axis.

Step 6: Interpretation of the $\bar{X}$-chart.

For the given problem, we have $k = 25$, $n = 4$ and $\sigma = 2.5$ ml.

1.4 STEPS INVOLVED IN THE CONSTRUCTION OF $\overline{X}$-CHART IN EXCEL 2007

You have already learnt the manual computation of $\overline{X}$-chart in Unit 2 of MSTE-001. Here we describe the procedure of constructing $\overline{X}$-chart for controlling the process mean in MS Excel 2007. In order to calculate the $\overline{X}$-chart control limits and to plot the control chart for the given data, we follow the steps given below:

Step 1: We enter each one of the k samples given in Table 1, in the MS Excel 2007 spreadsheet row-wise. For this example, if we start from Row 3 in the Excel sheet, the entry will go up to Row 27. For the given data, the spreadsheet will look as shown in Fig. 1.2.

	A	B	C	D	E
1		Juice Volume (in ml)			
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4
3	1	497.32	500.62	498.68	497.82
4	2	504.76	500.00	498.32	500.32
5	3	499.24	497.18	498.12	498.68
6	4	499.26	496.32	498.88	497.82
7	5	502.32	503.62	504.56	503.12
8	6	502.12	500.32	501.38	500.94
9	7	499.34	498.32	497.32	497.62
10	8	499.38	498.12	500.62	498.12
11	9	501.26	502.38	500.68	501.38
12	10	498.60	497.62	499.25	498.56
13	11	502.44	500.00	501.32	499.38
14	12	501.26	502.32	500.76	502.68
15	13	497.32	498.50	497.18	499.38
16	14	499.56	498.00	498.76	501.12
17	15	502.24	500.32	503.12	501.25
18	16	501.76	500.50	502.68	501.12
19	17	500.65	497.82	494.06	496.25
20	18	501.12	501.26	500.44	502.76
21	19	501.00	500.50	501.56	501.76

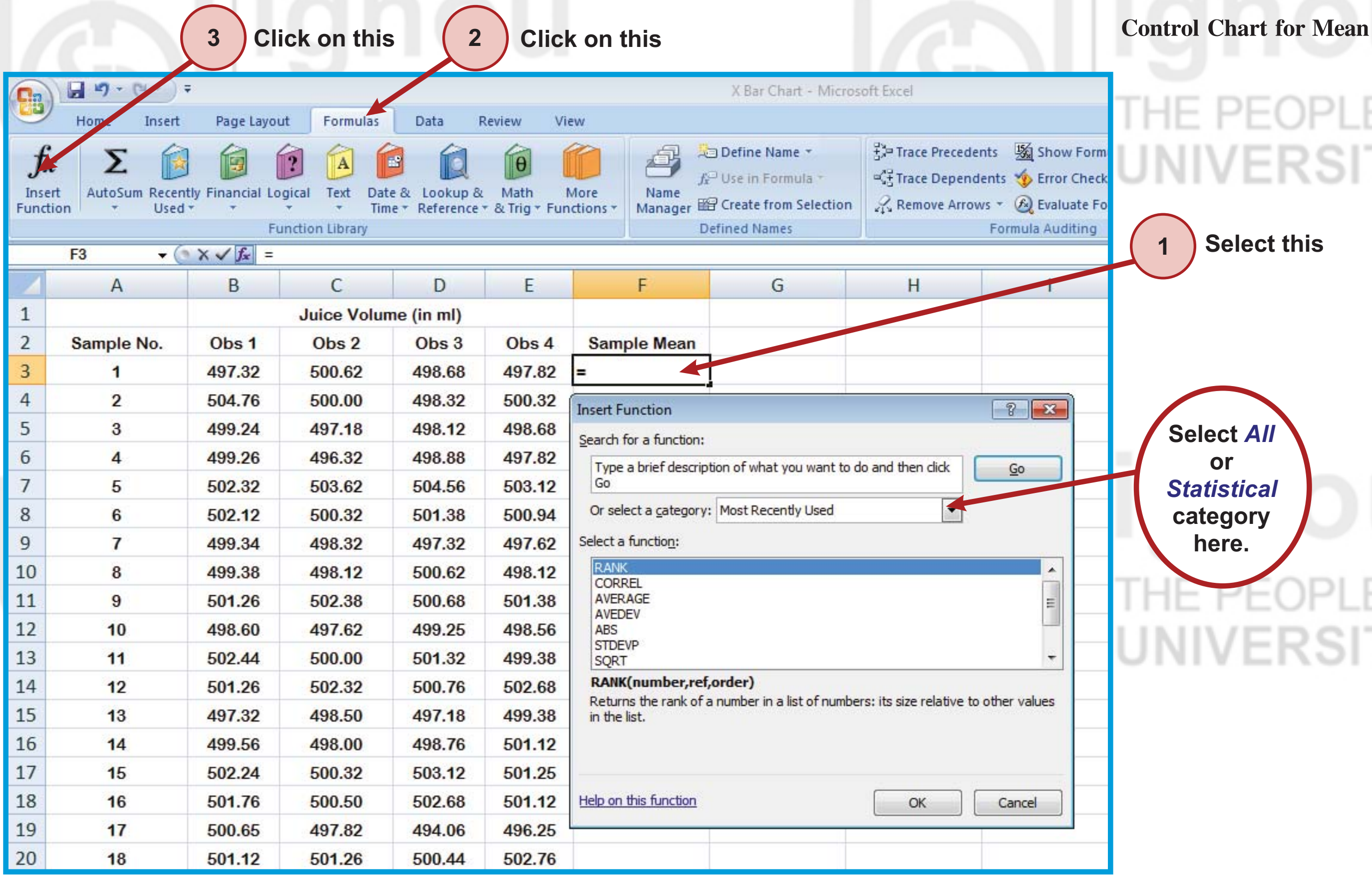
X Bar Chart

Fig. 1.2: Partial screenshot of the spreadsheet for the given data.

Step 2: We calculate the mean of each sample, i.e.,

$$\overline{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n}$$

For each sample of 4 observations of the volume of the filled bottle, we create a column for the sample mean. Here we are using Column F. We calculate the mean of 4 observations of the volume of bottle filled for the first sample using the inbuilt function of MS Excel. For calculating $\overline{X}$ for the first sample, we select Cell F3, click on the ***Formulas*** tab on the menu ribbon and select ***Insert Function***. The dialog box shown in Fig. 1.3 will appear.



	Juice Volume (in ml)				
Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean
1	497.32	500.62	498.68	497.82	=
2	504.76	500.00	498.32	500.32	
3	499.24	497.18	498.12	498.68	
4	499.26	496.32	498.88	497.82	
5	502.32	503.62	504.56	503.12	
6	502.12	500.32	501.38	500.94	
7	499.34	498.32	497.32	497.62	
8	499.38	498.12	500.62	498.12	
9	501.26	502.38	500.68	501.38	
10	498.60	497.62	499.25	498.56	
11	502.44	500.00	501.32	499.38	
12	501.26	502.32	500.76	502.68	
13	497.32	498.50	497.18	499.38	
14	499.56	498.00	498.76	501.12	
15	502.24	500.32	503.12	501.25	
16	501.76	500.50	502.68	501.12	
17	500.65	497.82	494.06	496.25	
18	501.12	501.26	500.44	502.76	

Fig. 1.3

We further select ***All*** or ***Statistical*** category as shown in Fig. 1.3. As a result, the following dialog box appears. We select ***Average*** function and click on ***OK*** (Fig. 1.4).

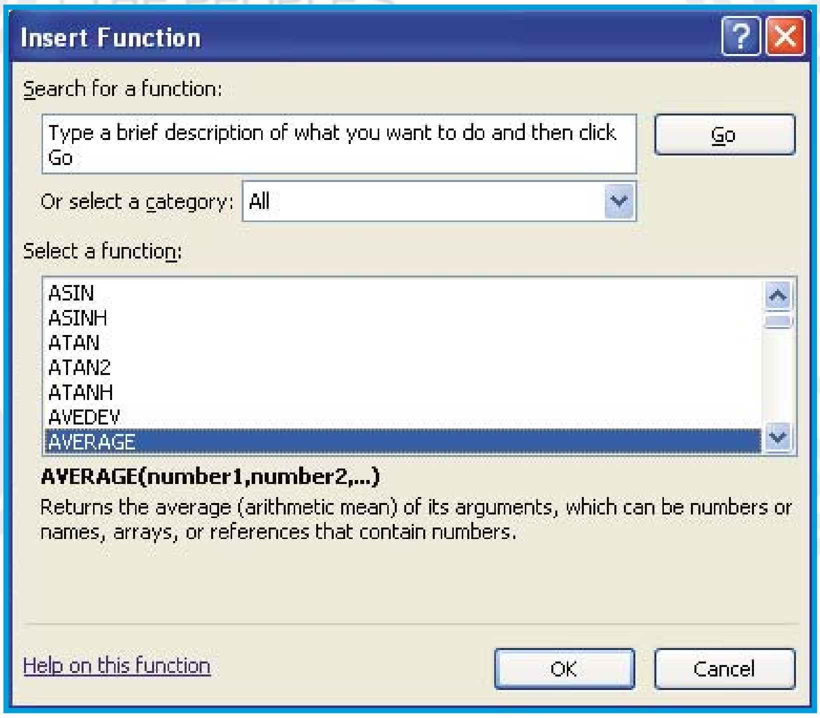


Fig. 1.4

Step 3: When we click on ***OK***, another dialog box appears as shown in Fig. 1.5. In this dialog box, data range can be taken by selecting observations of the sample given in Cells B3:E3. Then we click on ***OK*** or press the ***Enter*** key and get the average in Cell F3.

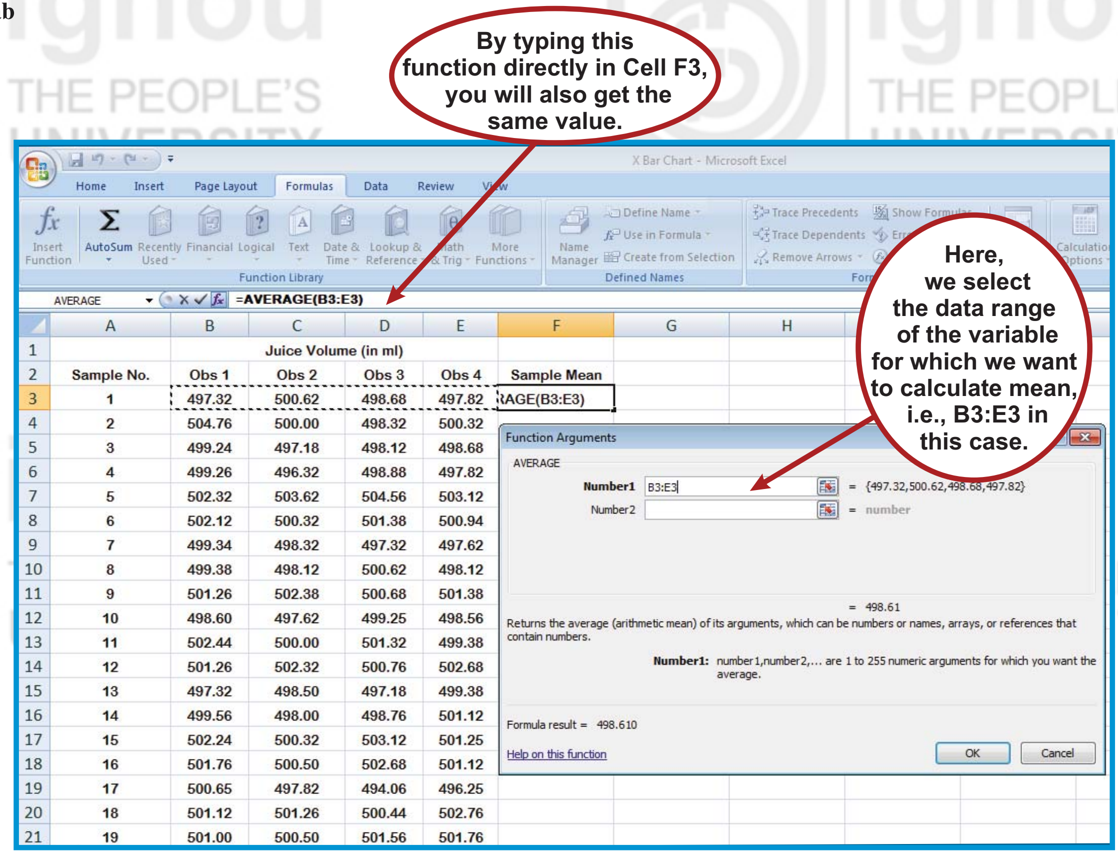


Fig. 1.5

Step 4: We put the cursor on the corner of Cell F3 and drag it down up to the last sample, i.e., Cell F27 as shown in Fig. 1.6.

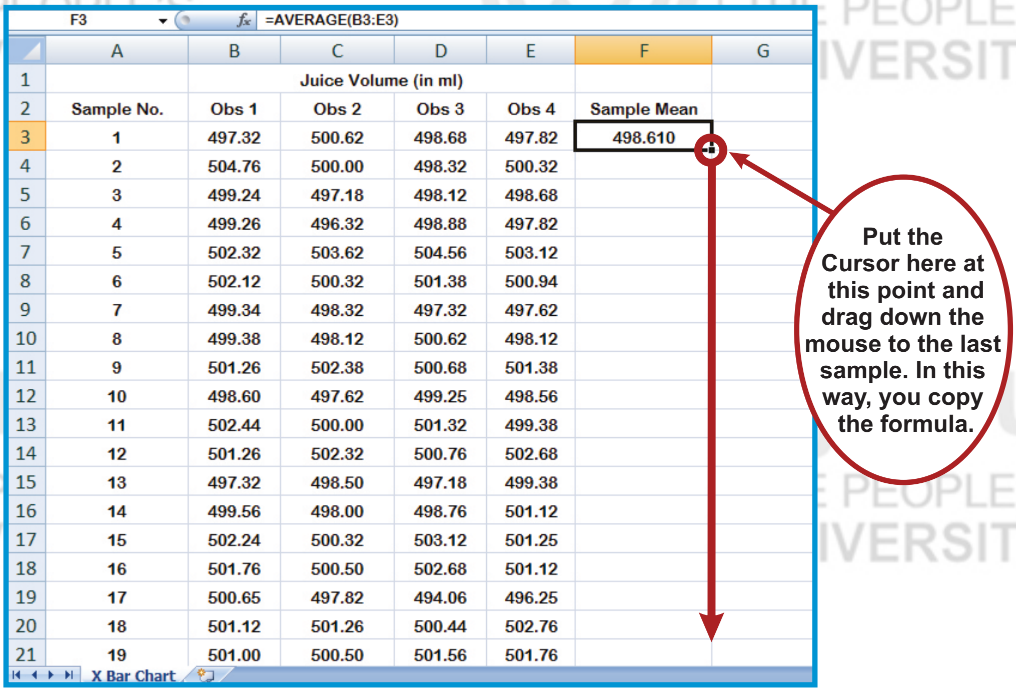


Fig. 1.6

Step 5: Step 4 will fill Column F up to the last sample, i.e., Cell F27. It will give the values of $\overline{X}$ for the remaining samples. The resulting spreadsheet is shown in Fig. 1.7.

	A	B	C	D	E	F
1				Juice Volume (in ml)		
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean
3	1	497.32	500.62	498.68	497.82	498.610
4	2	504.76	500.00	498.32	500.32	500.850
5	3	499.24	497.18	498.12	498.68	498.305
6	4	499.26	496.32	498.88	497.82	498.070
7	5	502.32	503.62	504.56	503.12	503.405
8	6	502.12	500.32	501.38	500.94	501.190
9	7	499.34	498.32	497.32	497.62	498.150
10	8	499.38	498.12	500.62	498.12	499.060
11	9	501.26	502.38	500.68	501.38	501.425
12	10	498.60	497.62	499.25	498.56	498.508
13	11	502.44	500.00	501.32	499.38	500.785
14	12	501.26	502.32	500.76	502.68	501.755
15	13	497.32	498.50	497.18	499.38	498.095
16	14	499.56	498.00	498.76	501.12	499.360
17	15	502.24	500.32	503.12	501.25	501.733
18	16	501.76	500.50	502.68	501.12	501.515
19	17	500.65	497.82	494.06	496.25	497.195
20	18	501.12	501.26	500.44	502.76	501.395
21	19	501.00	500.50	501.56	501.76	501.205

X Bar Chart

Fig. 1.7

Step 6: The sample means appear in Column F. To obtain the centre line and control limits for plotting the control chart, we first compute the grand average $\bar{\bar{X}}$ of the data, i.e., the average of 25 sample means (given in Cells F3:F27). For this, we use the method explained in Step 3 in Cell F28 as shown in Fig. 1.8.

F28 f_x =AVERAGE(F3:F27)

	D	E	F	G
28		$\bar{\bar{X}}$ =	499.918	
29				

Fig. 1.8

Step 7: We type the values of k, n and σ in Cells F29, F30 and F31, respectively, as shown in Fig. 1.9.

	E	F	G
28	$\bar{\bar{X}}$ =	499.918	
29	k =	25	
30	n =	4	
31	σ =	2.50	ml

Fig. 1.9

Step 8: We use equations (3) to (5) given in Sec. 1.3 for computing both the control limits and the centre line. Here we shall use Columns G, H and I for putting the values of centre line, upper and lower control limits, respectively. We calculate these as follows:

i) The formula for centre line is (CL) = $\overline{\overline{X}}$ and $\overline{\overline{X}}$ is given in Cell F28 (see Fig. 1.9). So we type "=$F$28" in Cell G3 and press ***Enter*** to get the value of centre line as shown in Fig. 1.10a.

ii) To calculate the upper control limit, the formula is

$$UCL = \overline{\overline{X}} + 3\frac{\sigma}{\sqrt{n}}$$

> The formula with dollar sign ($) is used for an absolute reference.

Since the values of $\overline{\overline{X}}$, n and σ are given in Cell F28, F30 and F31, respectively (see Fig. 1.9), we type "=$F$28+3*($F$31/(Sqrt ($F$30)))" in Cell H3 and press ***Enter***. Then we get the value of UCL in Cell H3 as shown in Fig. 1.10b.

iii) Similarly, we calculate the lower control limit $LCL = \overline{\overline{X}} - 3\frac{\sigma}{\sqrt{n}}$ by typing, "=$F$28–3*($F$31/(Sqrt ($F$30)))" in Cell I3 and pressing ***Enter***. We get the value of LCL as shown in Fig. 1.10c.

(a)

(b)

(c)

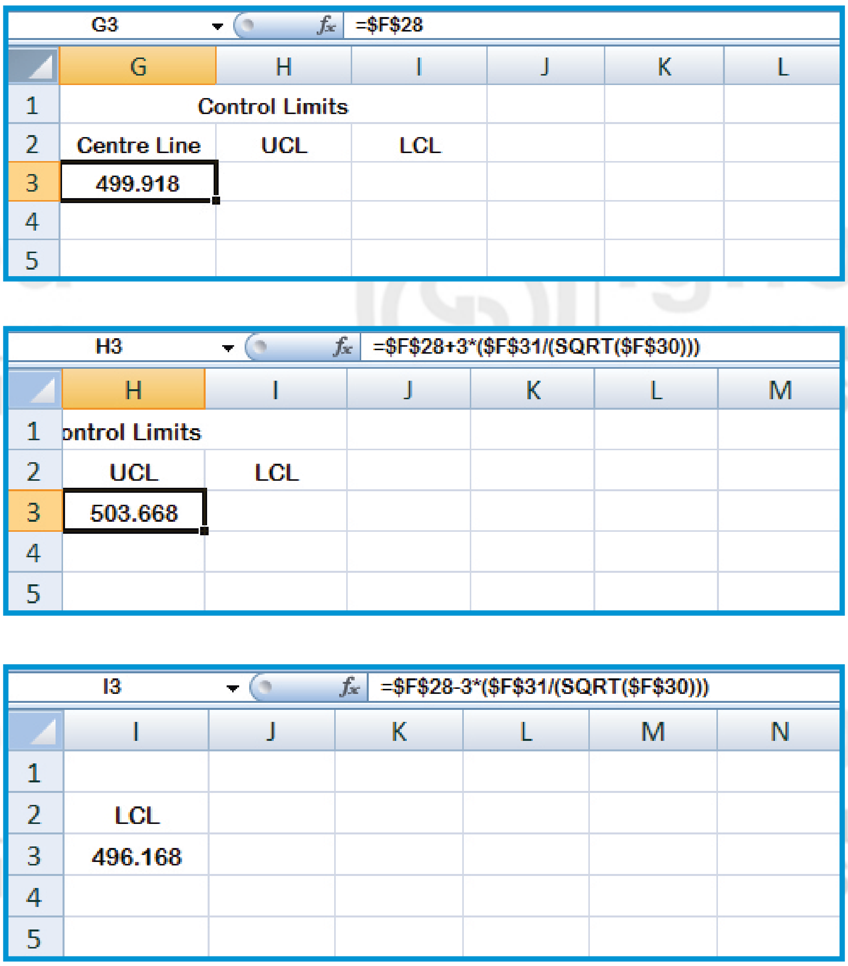


Fig. 1.10

Step 9: So we have calculated the CL, UCL and LCL in Cells G3, H3 and I3, respectively. We now have to plot these limits on the chart. For this purpose, we first select Cells G3:I3 and drag them down up to Row 27 as shown in Fig. 1.11a. After this activity, the Excel sheet should look like Fig. 1.11b.

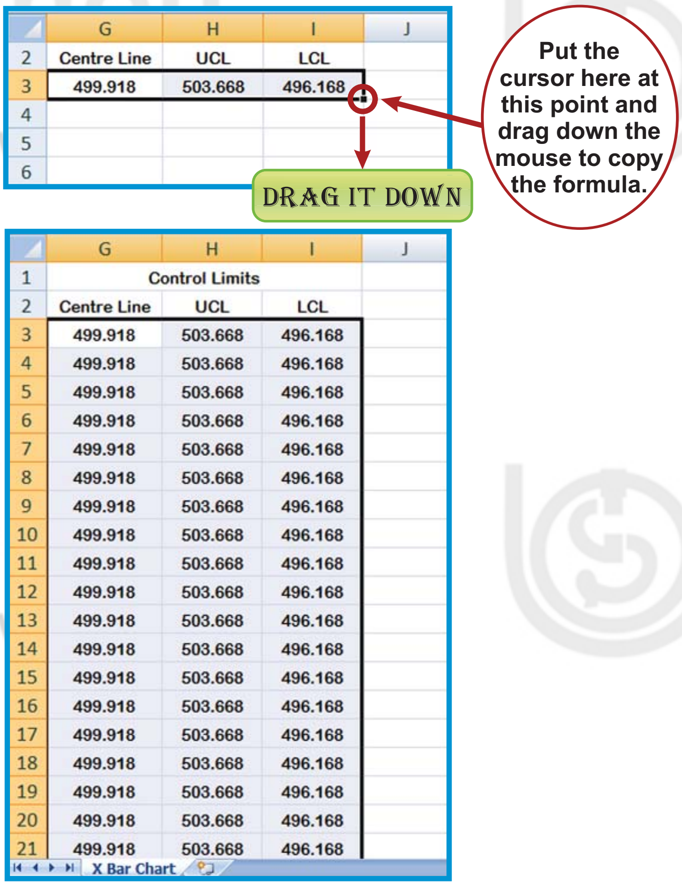


Fig. 1.11

> To scroll through all available chart types and chart subtypes, click on
>
> ✓ Any chart type in ***Chart*** group dialog box under ***Insert*** tab and then click ***All Chart Types***,
>
> ✓ Click the arrows to the right down corner of the ***Chart*** group, and then click the ones that you want to use.

Step 10: To obtain the $\bar{X}$-chart, we refer to Fig. 1.12. It means that we

1. select the Cells F2:I27,
2. click on the ***Insert*** tab,
3. click on the ***Line*** option in the ***Charts*** group, and
4. select a chart subtype that we want to use.

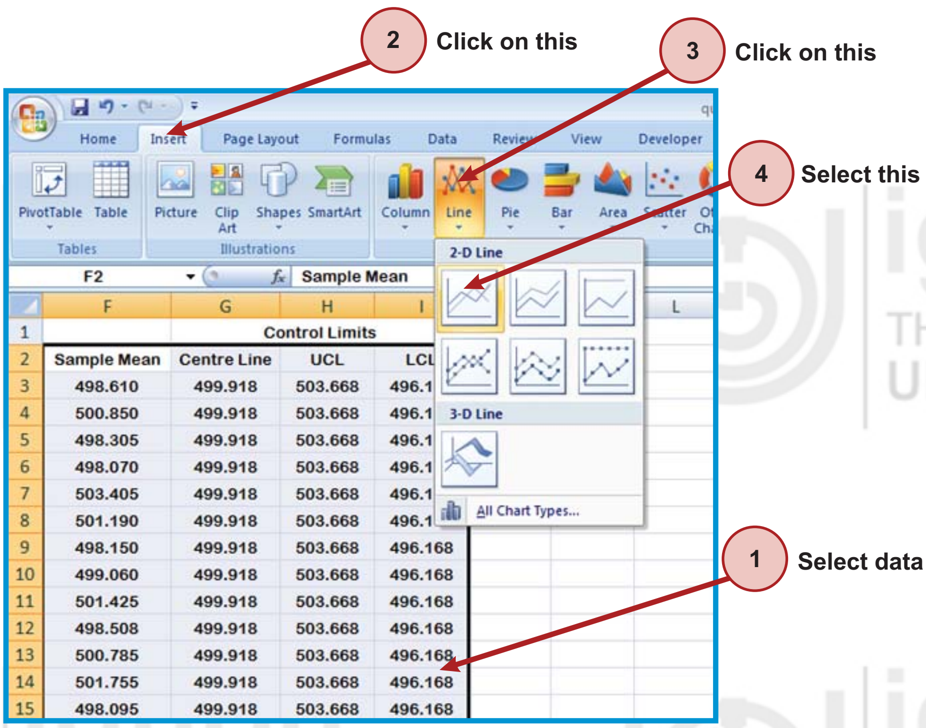


Fig. 1.12

Step 11: The values in Columns G, H and I provide the horizontal lines on the chart representing centre line, UCL and LCL, respectively. The values in Column F provide the averages for 25 samples on the chart. The resulting chart shown in Fig. 1.13 is called the $\overline{X}$-chart. Note that the sample means have been connected by straight line segments in Fig. 1.13.

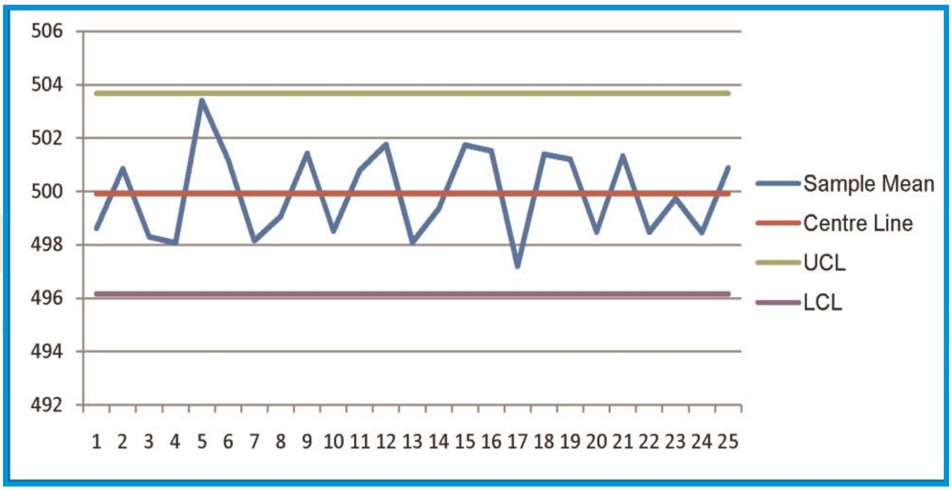


Fig. 1.13

Step 12: All that remains to be done is to format the chart the way you would like to present it. This is what you have already learnt in the Lab Session 3 of MSTL-001. For example, you may like to make the following changes:

- ✓ Eliminate the grid lines and the border around the chart.
- ✓ Change the UCL and LCL to dashed lines with no markers.
- ✓ Change the centre line to a bold solid line with no markers.
- ✓ Change the $\overline{X}$ data series to bold solid lines with large visible markers.
- ✓ Also change the fonts, axes, titles and background, etc. as desired.

We have formatted the chart as explained above. The resulting $\overline{X}$-chart is shown in Fig. 1.14.

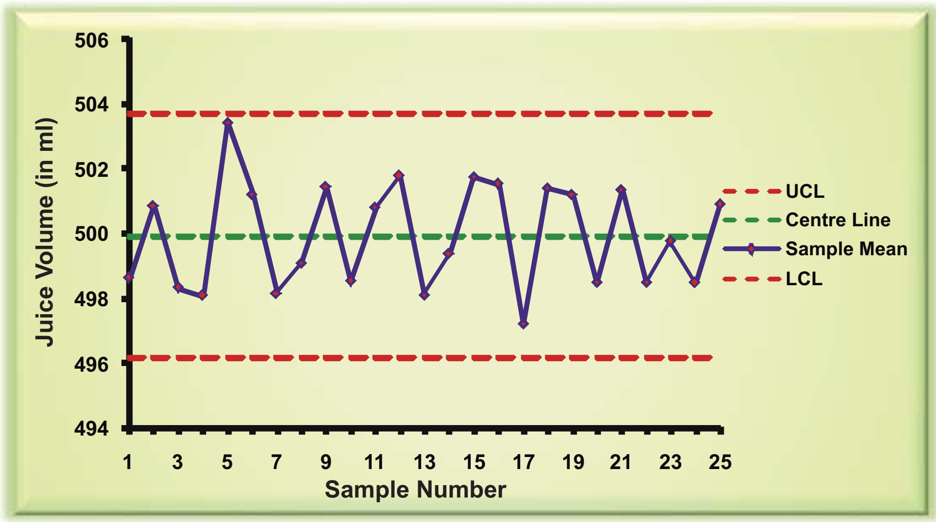


Fig. 1.14

1.5 INTERPRETATION

Now that you know how to make $\overline{X}$-chart in MS Excel, it is important for you to learn how to **interpret** it. If all points are within the control limits, we infer that the process is under statistical control. But, if any point goes outside the upper and lower control limits, the process is considered as out-of-control. In this kind of situation, the production process should be stopped and the causes of variations should be identified and eliminated.

The line segments in blue colour in Fig. 1.14 show the variability in the volume filled in the juice bottles. Since all these points lie between UCL and LCL, the process being monitored in the problem given in Sec. 1.2 is under statistical control (or stable). Here we may say that the process of bottling operation is free from assignable causes of variation and the variation is only under the influence of chance causes.

You should now apply this method to other problems for practice.

Activity

Construct the control charts for mean with the help of MS Excel 2007 and interpret the results for

A1) Example 3 given in Unit 2 of MSTE-001 if process standard deviation is 2.5.

A2) Exercise E6 given in Unit 2 of MSTE-001.

Match the results with the manual calculation done in Unit 2 of MSTE-001.

Continuous Assessment 1

Suppose a bulb manufacturing company wants to check whether the variation in the life of bulbs produced by a particular machine is due to chance causes or due to assignable causes. For this purpose, 12 samples, each of size 5, are selected and the life of each bulb is measured (in days). If the process variability is 0.25, develop the spreadsheet for the following data and construct a control chart for mean. Interpret it to infer whether the production is under control or not.

Table 2: Life of bulbs

Sample No.	Sample Observations				
1	42	65	75	78	87
2	42	45	68	72	90
3	19	24	80	81	81
4	36	54	89	77	84
5	42	51	57	59	78
6	51	74	75	78	132
7	60	60	72	95	138
8	18	20	27	42	60
9	15	30	39	62	84
10	69	109	113	118	153
11	64	90	93	109	112
12	61	78	94	109	136

Home Work: Do It Yourself

1) Follow the steps explained in Sec. 1.4 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 1" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute in your continuous assessment in the Laboratory.