4.5 Design of Sections for Flexure (Part IV)

This section covers the following topic.

- Magnel's Graphical Method

Notations

The variables used in this section are as follows.

A = area of cross section of member

c_t = distance of the top of the section from CGC

c_b = distance of the bottom of the section from CGC

e = eccentricity of CGS with respect to CGC

f_t = stress at the top of the section

f_b = stress at the bottom of the section.

$f_{cc,all}$ = allowable compressive stress in concrete

$f_{ct,all}$ = allowable tensile stress in concrete

I = moment of inertia of cross section of member

k_t = distance of top kern point from CGC

k_b = distance of bottom kern point from CGC

M_{SW} = moment due too self weight

M_T = total moment

P_0 = prestress at transfer after immediate losses

P_e = prestress at service after long term losses

r = radius of gyration, $r^2 = I/A$

Z_t = section modulus corresponding to top of the section = I/c_t

Z_b = section modulus corresponding to bottom of the section = I/c_b

η = ratio of prestressing forces = P_e/P_0

4.5.1 Magnel's Graphical Method

The determination of maximum and minimum eccentricities at the critical section helps in placing the CGS. But with different types of possible sections, the computations increase. The graphical method proposed by G. Magnel gives a visual interpretation of the equations involved.

There are essentially four stress conditions to be checked. These conditions are as follows.

- At transfer: $f_t \leq f_{ct,all}$ and $f_b \geq f_{cc,all}$
- At service: $f_t \geq f_{cc,all}$ and $f_b \leq f_{ct,all}$

The above expressions are algebraic inequalities where the stresses f_t and f_b are positive if tensile and negative if compressive. The allowable tensile stress $f_{ct,all}$ is assigned a positive value and the allowable compressive stress $f_{cc,all}$ is assigned a negative value. The allowable stresses are explained in the Section 1.5, Concrete (Part I).

It is to be noted that the values of $f_{cc,all}$ at transfer and at service are different. They are calculated based on the strength of concrete at transfer and at service, respectively.

Similarly, the values of $f_{ct,all}$ at transfer and at service can be different. As per **IS:1343 - 1980**, the values of $f_{ct,all}$ at transfer and service are of course same.

The stresses f_t and f_b in the four inequalities are expressed in terms of the initial prestressing force P_0, the eccentricity e at the critical section of the member, the section properties A, Z_t, Z_b, k_t, k_b and the load variables M_{sw} and M_T.

After transposition, $1/P_0$ is expressed in terms of e by linear inequality relationships.

For a selected section, these relationships are plotted in the $1/P_0$ versus e plane. The acceptable zone shows the possible combinations of $1/P_0$ and e that satisfy all the four inequality relationships. A combination of P_0 and e can be readily calculated from the acceptable zone.

The method is explained in a general form. For Type 1, Type 2 and Type 3 members, the value of allowable tensile stress ($f_{ct,all}$) is properly substituted. For Type 1 members, $f_{ct,all}$ = 0 N/mm^2.

At Transfer

The following sketch shows the variation of stress in concrete after the transfer of prestress and due to the self weight.

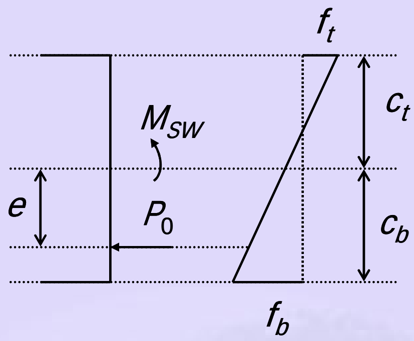


Figure 4-5.1 Stress profile in concrete at transfer

The stress at the top is calculated from P_0, e, M_{sw} as follows.

$$
\begin{aligned}
f_t &= -\frac{P_0}{A} + \frac{P_0 e c_t}{I} - \frac{M_{sw} c_t}{I} \\
&= \frac{P_0}{A}\left(-1 + \frac{e c_t}{r^2}\right) - \frac{M_{sw}}{Z_t} \\
&= \frac{P_0}{A}\left(-1 + \frac{e}{k_b}\right) - \frac{M_{sw}}{Z_t}
\end{aligned}
\qquad \textbf{(4-5.1)}
$$

The inequality relationship satisfying the stress at the top is expressed in terms of $1/P_0$ and e as follows.

$$f_t \le f_{ct,all}$$

$$\frac{P_0}{A}\left(-1 + \frac{e}{k_b}\right) - \frac{M_{sw}}{Z_t} \le f_{ct,all}$$

$$\text{or,}\quad \frac{1}{P_0} \ge \frac{(-1 + e/k_b)}{\left(f_{ct,all} + \frac{M_{sw}}{Z_t}\right)A} \qquad \textbf{(4-5.2)}$$

The following sketch shows the plot of inequality relationship. The straight line given by the above inequality is plotted in the $1/P_0$ versus e plane and the acceptable zone is shaded.

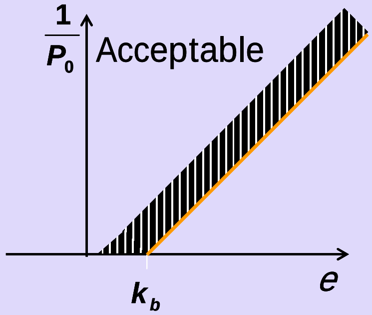


Figure 4-5.2 Plot based on stress at the top at transfer

The following expression relates the stress at the bottom with the load and section variables.

$$f_b = -\frac{P_0}{A} - \frac{P_0 e c_b}{I} + \frac{M_{sw} c_b}{I}$$

$$= -\frac{P_0}{A}\left(1 + \frac{e c_b}{r^2}\right) + \frac{M_{sw}}{Z_b}$$

$$= -\frac{P_0}{A}\left(1 + \frac{e}{k_t}\right) + \frac{M_{sw}}{Z_b} \qquad \textbf{(4-5.3)}$$

The inequality relationship satisfying the stress at the bottom is expressed as follows.

$$f_b \geq f_{cc,all}$$

$$-\frac{P_0}{A}\left(1 + \frac{e}{k_t}\right) + \frac{M_{sw}}{Z_b} \geq f_{cc,all}$$

$$\text{or,} \quad \frac{1}{P_0} \geq \frac{(1 + e/k_t)}{\left(-f_{cc,all} + \dfrac{M_{sw}}{Z_b}\right)A} \qquad \textbf{(4-5.4)}$$

The following sketch shows the plot of the inequality relationship.

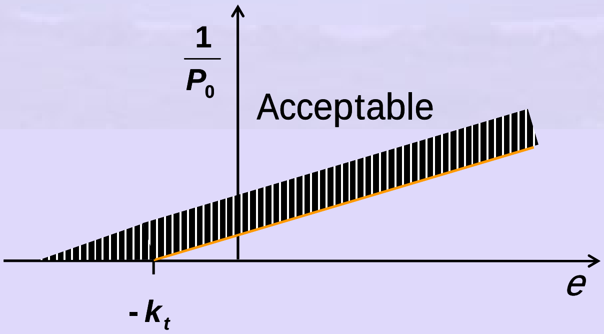


Figure 4-5.3 Plot based on stress at the bottom at transfer

At Service

The following sketch shows the variation of stress in concrete at service and due to the total moment.

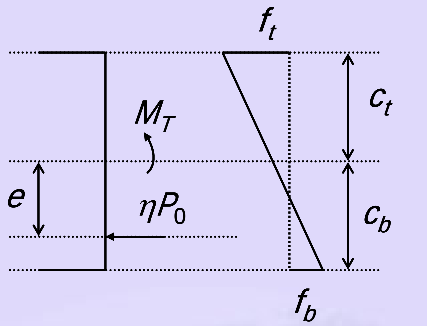


Figure 4-5.4 Stress profile in concrete at service

Here, P_e is expressed as ηP_0, where η is the ratio of effective prestress (P_e) and prestress at transfer (P_0).

The expression of the stress at the top is given below.

$$\begin{aligned} f_t &= -\frac{\eta P_0}{A} + \frac{\eta P_0 e c_t}{I} - \frac{M_T c_t}{I} \\ &= \frac{\eta P_0}{A}\left(-1 + \frac{e c_t}{r^2}\right) - \frac{M_T}{Z_t} \\ &= \frac{\eta P_0}{A}\left(-1 + \frac{e}{k_b}\right) - \frac{M_T}{Z_t} \end{aligned} \quad \textbf{(4-5.5)}$$

The inequality relationship satisfying the stress at the top is expressed as follows.

$$f_t \geq f_{cc,all}$$

$$\frac{\eta P_0}{A}\left(-1 + \frac{e}{k_b}\right) - \frac{M_T}{Z_t} \geq f_{cc,all}$$

$$\text{or, } \frac{1}{P_0} \leq \frac{\left(-1 + \dfrac{e}{k_b}\right)\eta}{\left(f_{cc,all} + \dfrac{M_T}{Z_t}\right)A} \quad \textbf{(4-5.6)}$$

The following sketch shows the plot of inequality relationship. The straight line given by the above inequality is again plotted in the $1/P_0$ versus e plane and the acceptable zone is shaded.

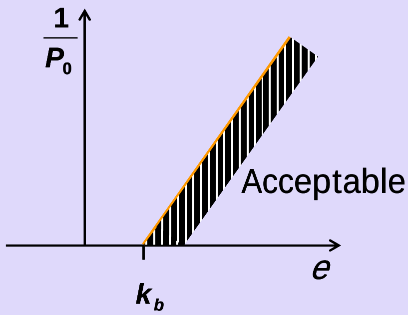


Figure 4-5.5 Plot based on stress at the top at service

The following expression relates the stress at the bottom with the load and section variables.

$$f_b = -\frac{\eta P_0}{A} - \frac{\eta P_0 e c_b}{I} + \frac{M_T c_b}{I}$$
$$= -\frac{\eta P_0}{A}\left(1 + \frac{e c_b}{r^2}\right) + \frac{M_T}{Z_b}$$
$$= -\frac{\eta P_0}{A}\left(1 + \frac{e}{k_t}\right) + \frac{M_T}{Z_b} \quad \textbf{(4-5.7)}$$

The inequality relationship is expressed as follows.

$$f_b \le f_{ct,all}$$
$$-\frac{\eta P_0}{A}\left(1 + \frac{e}{k_t}\right) + \frac{M_T}{Z_b} \le f_{ct,all}$$
$$\text{or, } \frac{1}{P_0} \le \frac{\left(1 + \frac{e}{k_t}\right)\eta}{\left(-f_{ct,all} + \frac{M_T}{Z_b}\right)A} \quad \textbf{(4-5.8)}$$

The following sketch shows the plot of the inequality relationship.

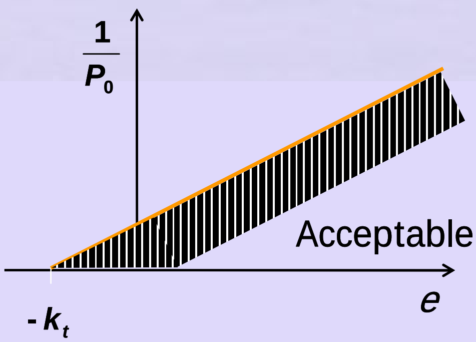


Figure 4-5.6 Plot based on stress at the bottom at service

Next, the four lines are plotted simultaneously. The common region is the acceptable zone.

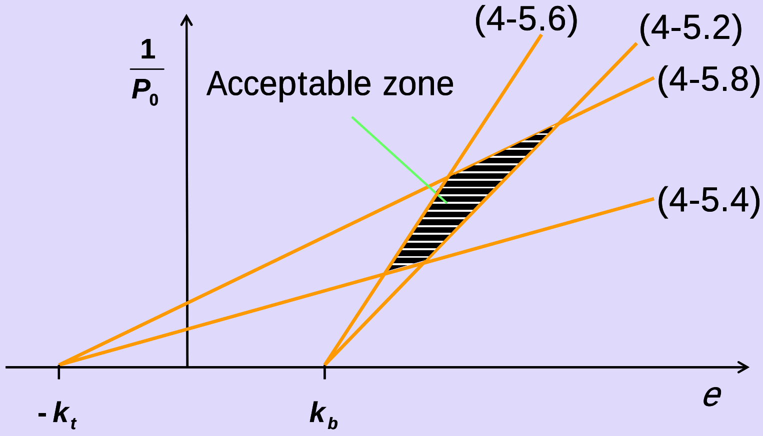


Figure 4-5.7 Acceptable zone

A combination of a trial section, prestressing force (P_0) and eccentricity (e) at the critical section, can be plotted in the form of the above graph. If the point lies within the acceptable zone, then the combination is valid.

The following problem illustrates the use of Magnel's graphical method.

Example 4-5.1

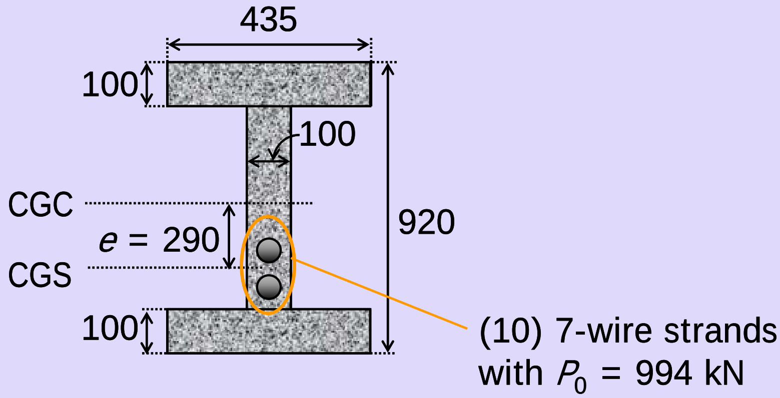
The section shown is designed as a Type 1 member with M_T = 435 kNm (including an estimated M_{SW} = 55 kNm). The height of the beam is restricted to 920 mm. The prestress at transfer f_{p0} = 1035 N/mm^2 and the prestress at service f_{pe} = 860 N/mm^2.

Based on the grade of concrete, the allowable compressive stresses are 12.5 N/mm^2 at transfer and 11.0 N/mm^2 at service.

The properties of the prestressing strands are given below.

Type of prestressing tendon : 7-wire strand
Nominal diameter = 12.8 mm
Nominal area = 99.3 mm^2

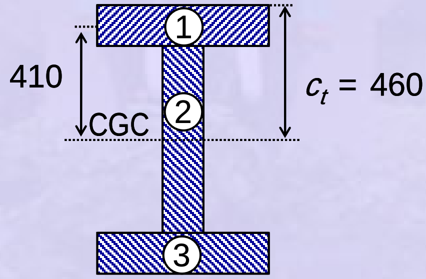
For the section, find the acceptable zone by Magnel's graphical method. Compare the designed values of eccentricity (e) and the inverse of prestressing force at transfer ($1/P_0$) with the acceptable zone.



Solution

A) Calculation of geometric properties

The section is symmetric about the horizontal axis. Hence, the CGC lies at mid depth. The section is divided into three rectangles for the computation of the geometric properties.



Values in mm.

Area of the section

$$
\begin{aligned}
A &= 2A_1 + A_2 \\
&= 2\times(435\times100) + (720\times100) \\
&= 159{,}000 \text{ mm}^2
\end{aligned}
$$

Moment of inertia of the section about axis through CGC

$$
\begin{aligned}
I &= 2I_1 + I_2 \\
&= 2\times\left[\frac{1}{12}\times435\times100^3 + (435\times100)\times410^2\right] + \frac{1}{12}\times100\times720^3 \\
&= 1.78\times10^{10} \text{ mm}^4
\end{aligned}
$$

Square of the radius of gyration

$$r^2 = \frac{I}{A}$$
$$= \frac{1.7808 \times 10^{10}}{159{,}000}$$
$$= 112{,}000\ \text{mm}^2$$

Section moduli

$$Z_b = Z_t = \frac{I}{c_t} = 38{,}712{,}174\ \text{mm}^3$$

Kern levels

$$k_b = k_t = \frac{r^2}{c_t} = 243.5\ \text{mm}$$

B) Calculation of the inequality relationships of Magnel's graphical method

Ratio of effective prestress and prestress at transfer

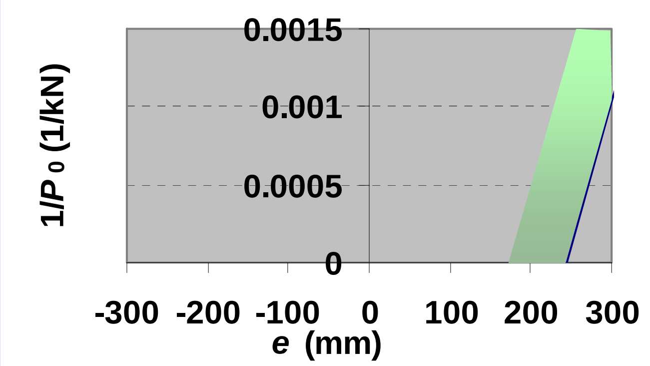
$$\eta = \frac{P_e}{P_0}$$
$$= \frac{f_{pe}}{f_{p0}} = \frac{860}{1035}$$
$$= 0.83$$

At Transfer

$f_t \le f_{ct,all}$
$$\frac{1}{P_0} \ge \frac{(-1 + e/k_b)}{\left(f_{ct,all} + \frac{M_{sw}}{Z_t}\right)A}$$

$$\frac{1}{P_0} \ge \frac{-1 + e/243.5}{\left(0 + \frac{55 \times 10^6}{38{,}712{,}174}\right) \times 159{,}000}$$
$$\ge \frac{1}{225{,}897.9}\left(-1 + \frac{e}{243.5}\right)$$

The relationship is plotted in the following graph.



At Transfer

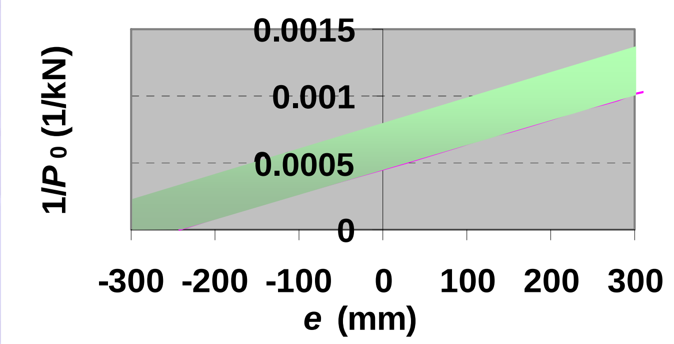
$f_b \geq f_{cc,all}$

$$\frac{1}{P_0} \geq \frac{(1+e/k_t)}{\left(-f_{cc,all} + \frac{M_{sw}}{Z_b}\right)A}$$

$$\frac{1}{P_0} \geq \frac{1+e/243.5}{\left(12.5+\frac{55\times10^6}{38,712,174}\right)\times159,000}$$

$$\geq \frac{1}{2,213,397.9}\left(1+\frac{e}{243.5}\right)$$

The relationship is plotted in the following graph.



At Service

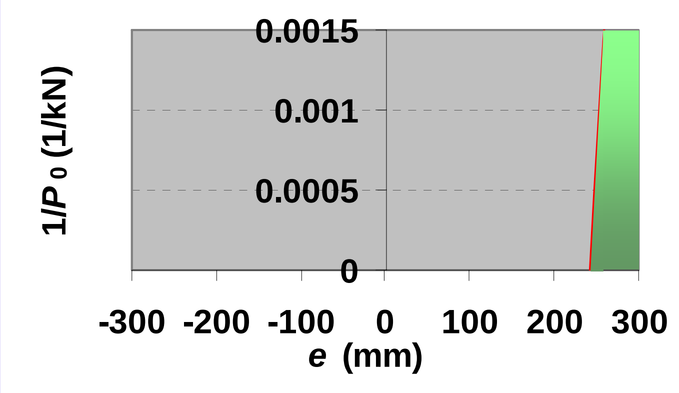
$f_t \geq f_{cc,all}$

$$\frac{1}{P_0} \leq \frac{\left(-1+\frac{e}{k_b}\right)\eta}{\left(f_{cc,all} + \frac{M_T}{Z_t}\right)A}$$

$$\frac{1}{P_0} \leq \frac{(-1+e/243.5)\times0.83}{\left(-11.0+\frac{435\times10^6}{38,712,174}\right)\times159,000}$$

$$\pounds \frac{1}{45,358.0}\left(-1+\frac{e}{243.5}\right)$$

The relationship is plotted in the following graph.



At Service

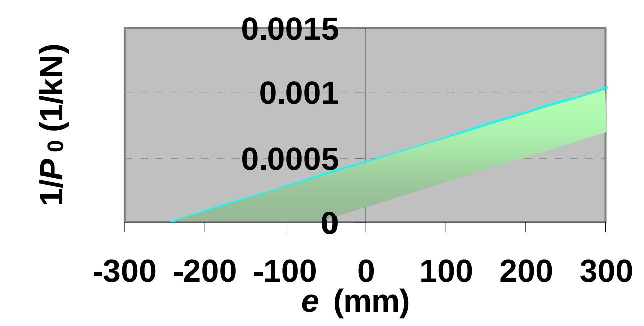
$f_b \le f_{ct,all}$

$$\frac{1}{P_0} \le \frac{\left(1+\dfrac{e}{k_t}\right)\eta}{\left(-f_{ct,all}+\dfrac{M_T}{Z_b}\right)A}$$

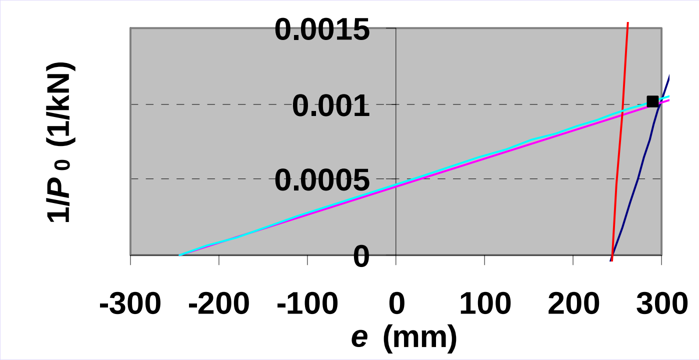
$$\frac{1}{P_0} \le \frac{(1+e/243.5)\times 0.83}{\left(0.0+\dfrac{435\times10^6}{38{,}712{,}174}\right)\times 159{,}000}$$

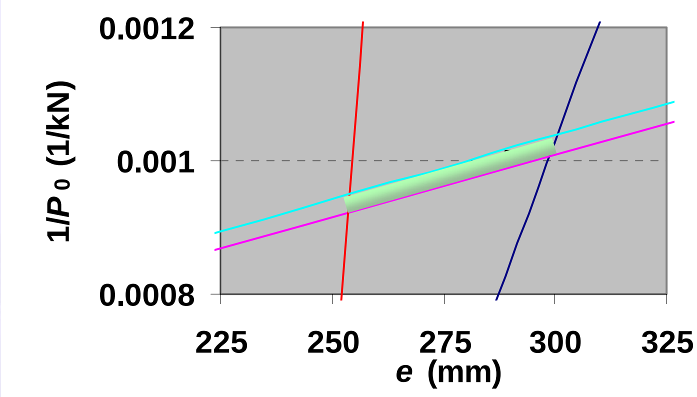
$$\le \frac{1}{2{,}152{,}587.1}\left(1+\frac{e}{243.5}\right)$$

The relationship is plotted in the following graph.



The four relationships are plotted in the following graph. The acceptable zone is shown. The zone is zoomed in the next graph.





The calculated values of e and $1/P_0$ for the Type 1 section are as follows.

e = 290 mm

$1/P_0$ = 1/(994 kN) = 0.001kN^{-1}.

The solution of the design is shown in the graphs. It lies in the acceptable zone.