3.1 Analysis of Members under Axial Load

This section covers the following topics.

- Introduction
- Analysis at Transfer
- Analysis at Service Loads
- Analysis of Ultimate Strength
- Analysis of Behaviour

Notations

Geometric Properties

A prestressed axial member may also have non-prestressed reinforcement to carry the axial force. This type of members is called partially prestressed members. The commonly used geometric properties of a prestressed member with non-prestressed reinforcement are defined as follows.

A = gross cross-sectional area

A_c = area of concrete

A_s = area of non-prestressed reinforcement

A_p = area of prestressing tendons

A_t = transformed area of the section

$= A_c + (E_s/ E_c)\, A_s + (E_p/ E_c)\, A_p$

The following figure shows the commonly used areas of a prestressed member with non-prestressed reinforcement.

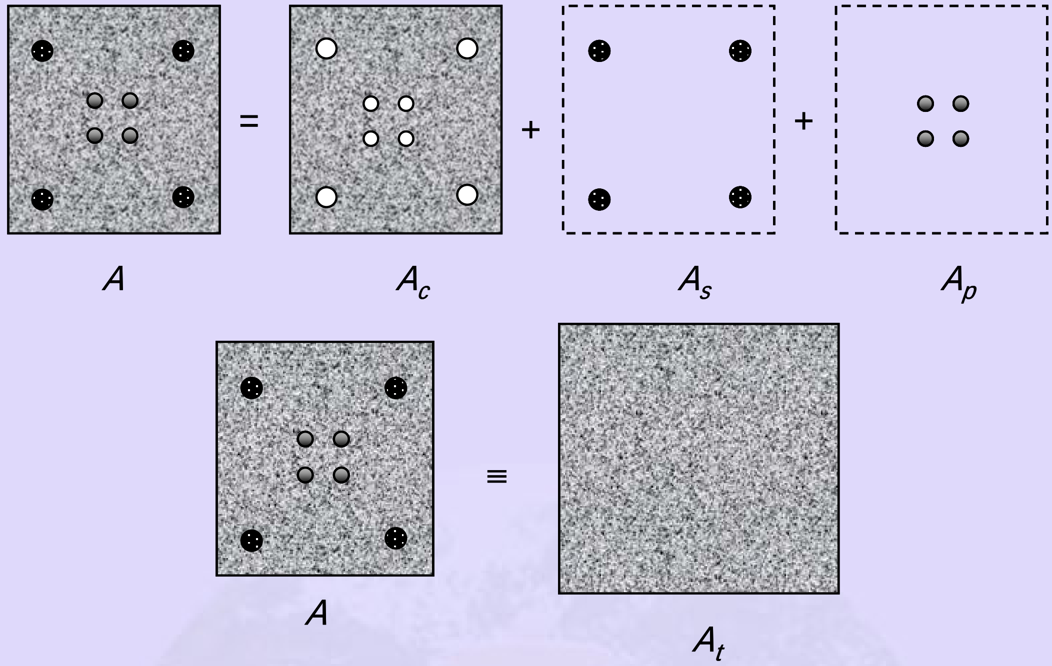

Figure 3-1.1 Areas for a prestressed member with non-prestressed reinforcement

3.1.1 Introduction

The study of members under axial load gives an insight of the behaviour of a prestressed member as compared to an equivalent non-prestressed reinforced concrete member. Prestressed members under axial loads only, are uncommon. Members such as hangers and ties are subjected to axial tension. Members such as piles may have bending moment along with axial compression or tension. In this section, no eccentricity of the CGS with respect to CGC is considered. The definitions of CGS and CGC are provided in Section 2.1, Losses in Prestress (Part I). The following figure shows members under axial loads.

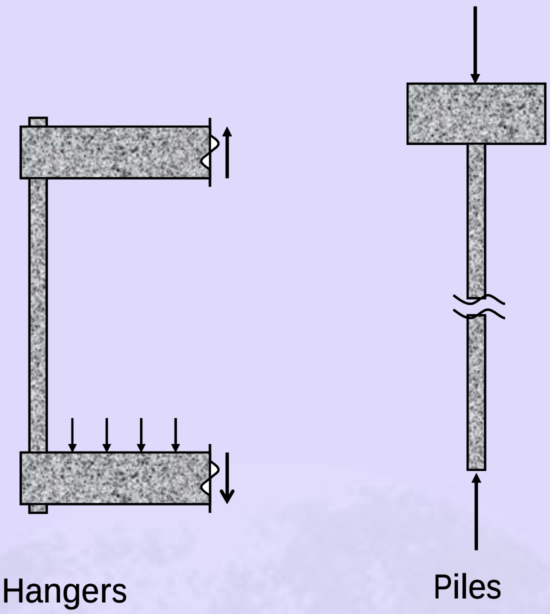

Figure 3-1.2 Members under axial load

The analysis of members refers to the evaluation of the following.

1) Permissible prestress based on allowable stresses **at transfer**.
2) Stresses under **service loads**. These are compared with allowable stresses under service conditions.
3) Ultimate strength. This is compared with the demand under **factored loads**.
4) The entire axial load versus deformation behaviour.

The stages for loading are explained in Section 1.2, Advantages and Types of Prestressing

3.1.2 Analysis at Transfer

The stress in the concrete (f_c) in a member without non-prestressed reinforcement can be calculated as follows.

$$f_c = -\frac{P_0}{A_c} \quad \textbf{(3-1.1)}$$

Here,

P_0 = prestress at transfer after short-term losses.

In presence of non-prestressed reinforcement, the stress in the concrete can be calculated as follows.

$$f_c = -\frac{P_0}{A_c + (E_s/E_c)A_s} \quad \textbf{(3-1.2)}$$

The permissible prestress is determined based on f_c to be within the allowable stress at transfer.

3.1.3 Analysis at Service Loads

The stresses in concrete in a member without non-prestressed reinforcement can be calculated as follows.

$$f_c = -\frac{P_e}{A_c} \pm \frac{P}{A_t} \quad \textbf{(3-1.3)}$$

Here,

P = external axial force (In the equation, + for tensile force and vice versa.)

P_e = effective prestress.

If there is non-prestressed reinforcement, A_c is to be substituted by (A_c + (E_s/E_c) A_s) and A_t is to be calculated including A_s.

The value of f_c should be within the allowable stress under service conditions.

3.1.4 Analysis of Ultimate Strength

The ultimate tensile strength of a section (P_{uR}) can be calculated as per **Clause 22.3, IS:1343 - 1980**.

In absence of non-prestressed reinforcement,

$$P_{uR} = 0.87 f_{Pk} A_p \quad \textbf{(3-1.4a)}$$

In presence of non-prestressed reinforcement,

$$P_{uR} = 0.87 f_y A_s + 0.87 f_{Pk} A_p \quad \textbf{(3-1.4b)}$$

In the previous equations,

f_y = characteristic yield stress for non-prestressed reinforcement with mild steel bars

= characteristic 0.2% proof stress for non-prestressed reinforcement with high yield strength deformed bars.

f_{pk} = characteristic tensile strength of prestressing tendons.

The ultimate tensile strength should be greater than the demand due to factored loads.

The ultimate compressive strength of a section (P_{uR}) can be calculated in presence of moments by the use of **interaction diagrams**. For a member under compression with minimum eccentricity, the ultimate strength is given as follows. Here, the contribution of prestressing steel is neglected.

$$P_{uR} = 0.4\, f_{ck} A_c + 0.67\, f_y\, A_s \qquad \textbf{(3-1.5)}$$

3.1.5 Analysis of Behaviour

The analysis of behaviour refers to the determination of the complete axial load versus deformation behaviour. The analyses at transfer, under service loads and for ultimate strength correspond to three instants in the above behaviour.

The analysis involves three principles of mechanics (Reference: Collins, M. P. and Mitchell, D., *Prestressed Concrete Structures*, Prentice-Hall, Inc., 1991).

1) **Equilibrium** of internal forces with the external loads at any point of the load versus deformation behaviour. The internal forces in concrete and steel are evaluated based on the respective strains, cross-sectional areas and the constitutive relationships.
2) **Compatibility** of the strains in concrete and in steel for bonded tendons. This assumes a perfect bond between the two materials. For unbonded tendons, the compatibility is in terms of total deformation.
3) **Constitutive relationships** relating the stresses and the strains in the materials. The relationships are developed based on the material properties.

Equilibrium Equation

At any instant, the equilibrium is given by the following equation.

$$P = A_c f_c + A_s f_s + A_p f_p \quad \textbf{(3-1.6)}$$

Here,

f_c = stress in concrete

f_s = stress in non-prestressed reinforcement

f_p = stress in prestressed tendons

P = axial force.

Compatibility Equations

For non-prestressed reinforcement

$$\varepsilon_s = \varepsilon_c \quad \textbf{(3-1.7)}$$

For prestressed tendons

$$\varepsilon_p = \varepsilon_c + \Delta\varepsilon_p \quad \textbf{(3-1.8)}$$

Here,

ε_c = strain in concrete at the level of the steel

ε_s = strain in non-prestressed reinforcement

ε_p = strain in prestressed tendons

$\Delta\varepsilon_p$ = strain difference in prestressed tendons with adjacent concrete

The strain difference ($\Delta\varepsilon_p$) is the strain in the prestressed tendons when the concrete has zero strain (ε_c = 0). This occurs when the strain due to the external tensile axial load balances the compressive strain due to prestress. At any load stage,

$$\Delta\varepsilon_p = \varepsilon_{pe} - \varepsilon_{ce} \quad \textbf{(3-1.9)}$$

Here,

ε_{pe} = strain in tendons due to P_e, the prestress at service

ε_{ce} = strain in concrete due to P_e.

The strain difference is further explained in Section 3.4, Analysis of Member under Flexure (Part III).

Constitutive Relationships

The constitutive relationships can be expressed in the following forms based on the material stress-strain curves shown in Section 1.6, Concrete (Part II), and Section 1.7, Prestressing Steel.

For concrete under compression

$$f_c = F_1 (\varepsilon_c) \qquad \textbf{(3-1.10)}$$

For prestressing steel

$$f_p = F_2 (\varepsilon_p) \qquad \textbf{(3-1.11)}$$

For reinforcing steel

$$f_s = F_3 (\varepsilon_s) \qquad \textbf{(3-1.12)}$$

The stress versus strain curve for concrete is shown below. The first and third quadrants represent the behaviour under tension and compression, respectively.

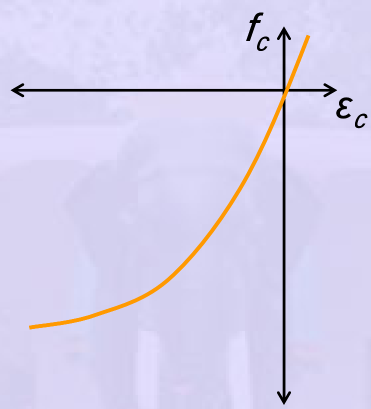

Figure 3-1.3 Stress versus strain for concrete

The stress versus strain curve for prestressing steel is as shown below.

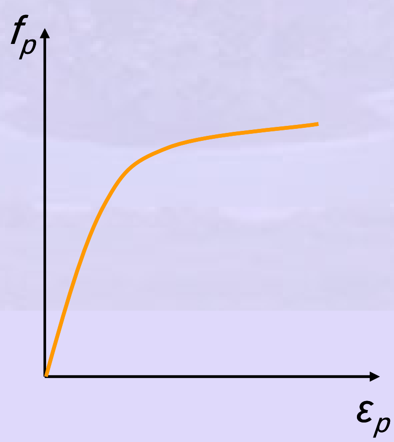

Figure 3-1.4 Stress versus strain for prestressing steel

The following stress versus strain curve is for reinforcing steel.

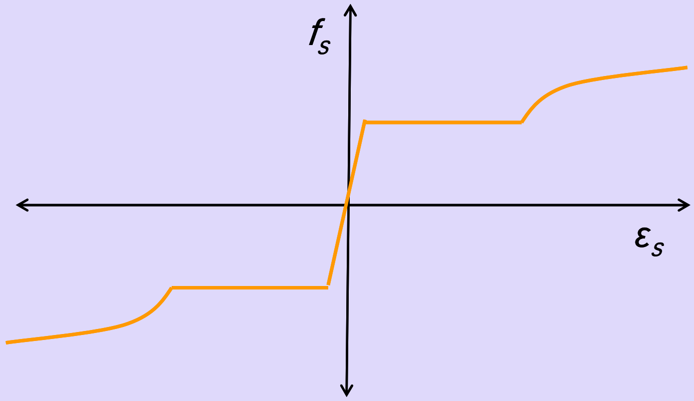

Figure 3-1.5 Stress versus strain for reinforcing steel

The equilibrium and compatibility equations and the constitutive relationships can be solved to develop the axial force versus deformation curve. The deformation can be calculated as $\varepsilon_c L$, where L is the length of the member.

The following plot shows the axial force versus deformation curves for prestressed and non-prestressed sections. The two sections are equivalent in their ultimate tensile strengths.

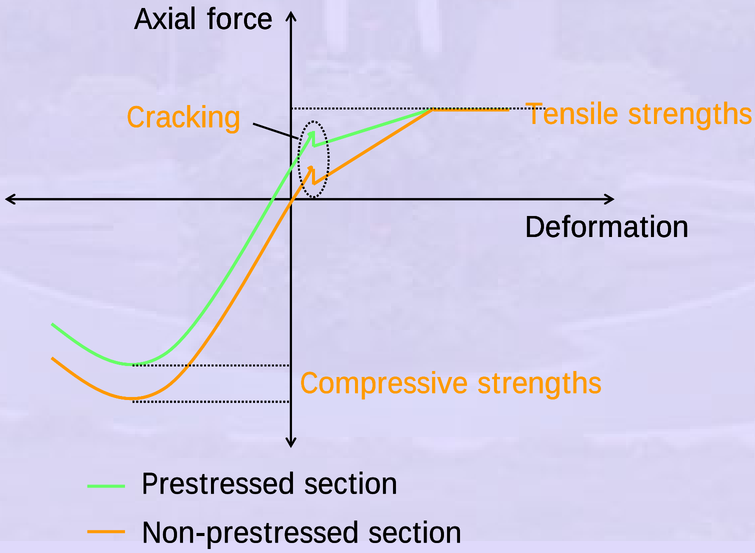

Figure 3-1.6 Axial force versus deformation curves

From the previous plot, the following can be inferred.

1) Prestressing increases the cracking load.
2) Prestressing shifts the curve from the origin.
   - For the prestressed member, there is a compressive deformation in absence of external axial force.
   - A certain amount of external force is required to decompress the member.

3) For a given tensile load, the deformation of the prestressed member is smaller.
   - Prestressing reduces deformation at service loads.
4) For a given compressive load, the deformation of the prestressed member is larger.
   - Prestressing is detrimental for the response under compression.
5) The compressive strength of the prestressed member is lower.
   - Prestressing is detrimental for the compressive strength.
6) For a partially prestressed section with the same ultimate strength, the axial load versus deformation curve will lie in between the curves for prestressed and non-prestressed sections.

The above conclusions are generic for prestressed members.